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SUMMARY

The new orbit determination program PROP can use range and range rate observations as well as observations of direction. The orbit of Ogo 2 (1965-81A) has been determined at epoch 1965 November 10.0, using Minitrack observations and Goddard Range and Range Rate observations. The results show reasonable agreement between observations from the two systems and suggest that random errors in the range and range rate observations are of the order claimed.

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CONTENTS

1 INTRODUCTION

Past determinations of earth-satellite orbits at the Royal Aircraft Establishment have been restricted to the use of directional observations only, i.e. to observations of right ascension and declination, azimuth and elevation, or a pair of direction cosines. During the last two years, however, a new computer program has been developed which permits the use of other types of observation, including range, range rate and the rates of change of directional data.

The first application of the new program has been to the orbit of Ogo 2 (1965-81A) over a period of four days in November 1965. This satellite was tracked by two NASA systems of stations - the Minitrack network and the Goddard Range and Range Rate System. Thus an opportunity was available to test the ability of the new program to deal with observations of ρ and $\dot{\rho}$ (range and range rate) and to see how the accuracy of orbital parameters determined from Goddard ρ and $\dot{\rho}$ observations compared with their accuracy when determined from Minitrack observations. There was also the opportunity to form some assessment of the accuracy of the ρ and $\dot{\rho}$ observations themselves.

2 THE GODDARD RANGE AND RANGE RATE SYSTEM

The GRARR System was developed to supplement the Minitrack network, which consists of about a dozen interferometer stations. Minitrack observations are very suitable for close-earth satellites, but for satellites at greater distances they suffer from a weakness which is inherent in all directional observations: viz. that a fixed error in angle generates an error cone and hence is equivalent to a positional error which is proportional to range.

The GRARR System consists of stations at Carnarvon (Australia), Rosman (North Carolina), Tananarive (Malagasy), Fairbanks (Alaska) and Santiago (Chile). The first three were constructed first, and data from the other two have not been received. Each station operates at either VHF or S-Band and is capable of measuring range and/or range rate. (Stations can also measure direction, but errors may exceed 0.1°, so that this is not normally very useful.) The Ogo 2 observations received from NASA used the S-Band mode of operation, which is the more accurate and is likely to be the more extensively used. The VHF antenna is required even in the S-Band mode,

however, since it is responsible for initial acquisition, by locking onto the 136 MHz Minitrack beacon in the satellite. The S-Band antenna is initially slaved to the VHF antenna but, after acquisition by the S-Band receiver, it adopts an autotrack mode of operation.

Range is measured by use of a ground-transmitted 100 kHz sidetone frequency and a series of lower frequency tones to resolve ambiguities. Range rate is measured by counting cycles of the two-way doppler shift.

The performance objectives for the GRARR System specify[1] S-Band accuracies (rms tolerances) of 15 m in ρ and 0.1 m/s in $\dot{\rho}$.

3 OGO 2 OBSERVATIONS

Minitrack observations on punched cards have been received (at R.A.E.) for the period November 9-11 1965, and GRARR observations, also on punched cards, for the period November 7-12 1965. It was decided to work with about 60 of each type of observation, covering the period November 8-11, and to refer orbital parameters to a single epoch, namely 1965 November 10.0.

The observations were weighted in the orbit determination program, using the following a priori estimates of error (s.d.): for the Minitrack observations, 0.00029 in the direction cosines, equivalent to about 1' in angular measure, as suggested by previous R.A.E. experience[2]; for the GRARR observations, 150 m in ρ and $\frac{1}{2}$ m/s in $\dot{\rho}$, assessing these much more conservatively than the performance specification.

4 ORBITAL MODEL

The model for the new R.A.E. orbit determination program (known as PROP) has been described by Merson[3]. It takes account of short-periodic, secular and long-periodic perturbations due to the zonal harmonics of the earth's gravitational field and to atmospheric drag.

There is some flexibility in the choice of the particular set of orbital parameters to be fitted by PROP. The following set was chosen: e (eccentricity), i (inclination), Ω (right ascension of the node), ω (argument of perigee), M_0 (mean anomaly), M_1 and M_2; these are assumed to be epoch values in each case. The last two parameters are such that the mean anomaly at time t (relative to epoch) is given - apart from short-periodic and long-periodic perturbations - by

$$M = M_0 + M_1 t + M_2 t^2 \quad .$$

Thus M_1 is the mean motion at epoch; it is related to a, the semi-major axis, by

$$a = (\mu/M_1^2)^{1/3} - \tfrac{1}{2} J_2 R^2 (\mu/M_1^2)^{-1/3} (1 - 1\tfrac{1}{2} \sin^2 i)(1 - e^2)^{-1\frac{1}{2}} \quad .$$

Here μ, J_2 and R are familiar earth constants, for which the following values have been taken:

$$\mu = 398602 \text{ km}^3/\text{s}^2, \qquad J_2 = 10^{-6} \times 1082.68, \qquad R = 6378.163 \text{ km} \quad .$$

5 RESULTS

The determination of orbital parameters at the selected epoch was carried out three times: first, using 59 Minitrack observations; second, using 27 range and 28 range rate observations; finally, using all 114 observations together. The results are summarized in Table 1. It is to be remembered that there are only 7 independent parameters; the semi-major axis a is a derived parameter.

Table 1

	Case 1	Case 2	Case 3
a (km)	7344.8382 ±0.0008	7344.8427 ±0.0002	7344.8414 ±0.0002
e	0.075948 ±4 × 10^{-6}	0.075950 ±2 × 10^{-6}	0.075947 ±2 × 10^{-6}
i (deg)	87.3574 ±0.0007	87.3591 ±0.0005	87.3571 ±0.0005
Ω (deg)	-84.5244 ±0.0006	-84.5241 ±0.0002	-84.5245 ±0.0003
ω (deg)	92.5875 ±0.0052	92.6195 ±0.0018	92.6105 ±0.0024
M_0 (deg)	-73.6895 ±0.0048	-73.7187 ±0.0018	-73.7110 ±0.0022
M_1 (deg/d)	4966.6788 ±0.0008	4966.6742 ±0.0002	4966.6756 ±0.0002
M_2 (deg/d^2)	0.0022 ±0.0006	0.0024 ±0.0002	0.0041 ±0.0002
rejections	6 out of 59	4 out of 55	11 out of 114
ε	1.28	0.56	1.06

Some explanation is necessary of the last two rows of Table 1.

The quantity ε, known as the standard deviation of an observation of unit weight, is a measure of the goodness of fit. It is given by $\varepsilon = \sqrt{\Sigma/D_f}$, where Σ is the sum of the squares of the final weighted

residuals and D_f is the number of degrees of freedom. If the accuracies of the observations have been correctly assessed, and if the errors in the observations are random and normally distributed, then the expected value of ε is 1.0. The standard deviations given for the orbital parameters all include the appropriate ε as a factor; this is why some of the standard deviations for the third orbital determination, for which all the observations were included, are greater than the corresponding values for the second determination, for which only about half the observations were used but for which ε was only 0.56.

The computer program has an automatic facility for rejecting any observation with a weighted residual which exceeds a certain quantity; in the three orbit determinations here the quantity concerned is 3ε. In the second determination the four observations rejected comprised two of range and two of range rate. Since in both the first two determinations there were several observations with weighted residuals in the vicinity of $\pm 3\varepsilon$ it is not surprising that the observations rejected in the third determination were not quite the same as those rejected in the earlier determinations - they comprised nine Minitrack, two range rate and no range observations. It was because the accepted observations were not just the sum of the accepted observations from the first two determinations that the orbital parameters do not always lie between the values given by the first two determinations.

6 DISCUSSION

An obvious conclusion from the values of ε in Table 1 is that the observations were wrongly weighted. The Minitrack observations appear somewhat less accurate than was assumed, while the GRARR observations are more accurate. Inspection of the residuals from the second orbit determination shows two things: first, that the average residual in ρ was only about 30 m, while that in $\dot{\rho}$ was about 0.3 m/s; second, that the random components were actually much less than this, a strong bias being present.

Biases of this order cannot be regarded as very serious. There are several possible explanations of them. The accuracy of the station coordinates is not known but it is unlikely to be better than 30 m; the position given for Rosman in fact differs by about $\frac{1}{2}$ km from that for an earlier survey. Errors in the orbital model, due in particular to the neglect of all but the dominant tesseral harmonic perturbation, will certainly exceed 30 m; over one pass of the satellite from any station these errors will appear as bias in the observations.

It is useful to consider the differences in the orbital parameters given by the two independent determinations. These differences may be expressed in terms of the standard deviations of the parameters, where we choose the first set of standard deviations since these are larger. We have

$$\delta a = 0.0045 \text{ km} = 5\tfrac{1}{2}\sigma, \quad \delta e = 2 \times 10^{-6} = \tfrac{1}{2}\sigma,$$

$$\delta i = 0^{\circ}.0017 = 2\tfrac{1}{2}\sigma, \quad \delta\Omega = 0^{\circ}.0003 = \tfrac{1}{2}\sigma,$$

$$\delta\omega = 0^{\circ}.0320 = 6\sigma, \quad \delta M_0 = 0^{\circ}.0292 = 6\sigma,$$

$$\delta M_1 = 0.0046^{\circ}/d = 5\tfrac{1}{2}\sigma \text{ and } \delta M_2 = 0.0002^{\circ}/d^2 = \tfrac{1}{3}\sigma \quad .$$

The existence of $5\frac{1}{2}\sigma$ and 6σ differences again indicates bias. For semi-major axis the bias may be due to error in the adopted value of μ. In the case of ω and M_0 it must be remarked that these parameters are very highly correlated, since the position of perigee is not well-determined.

7 CONCLUSIONS

The new R.A.E. computer program can deal satisfactorily with observations of range and range rate.

NASA observations from the Goddard Range and Range Rate System appear to have accuracies consistent with the performance objectives (15 m and 0.1 m/s), as far as random errors are concerned. For observations of Ogo 2 analysed by the R.A.E. program, however, the residuals are of the order 30 m and 0.3 m/s. The effective bias in these residuals can be attributed to limitations in the orbital model of the program; whether there are also actual biases in the observations cannot be decided.

For an orbit at an average height of about 1000 km orbital parameters obtained from the Minitrack network and parameters obtained from the GRARR System are of comparable accuracy. In a single comparison for the satellite Ogo 2 standard deviations for orbital parameters determined from Minitrack observations were only about half those for parameters determined from GRARR observations. For orbits at greater heights it is to be expected that the advantage of the GRARR System would become quite marked.

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